Written Exam for the M.Sc. in Economics Autumn 2012 (Fall Term)

Financial Econometrics A: Volatility Modelling

Final Exam: Masters course

Exam date: **FEBRUARY-2013**

3-hour open book exam.

Please note there are a total of 10 questions which should **all be replied to**. That is, 4 questions under *Question A*, and 6 under *Question B*.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question A:

Question A.1: Consider the Gaussian conditional standard deviation ARCH(1) model (STD-ARCH) for the returns x_t as given by,

$$x_t = \sigma_t z_t, \tag{A.1}$$

with $z_t \sim i.i.d.N(0,1)$ and,

$$\sigma_t = \omega + \alpha |x_{t-1}|, \quad \omega > 0, \quad \alpha \geq 0. \tag{A.2}$$

Derive a condition for x_t to be weakly mixing with finite variance.

Question A.2: With $l_t(\theta)$ denoting the log-likelihood contribution at time t for the STD-ARCH in terms of $\theta = (\omega, \alpha)$, show that if the true parameters α_0 and ω_0 satisfy, $\omega_0 = 1$ and $\alpha_0 = 2$ then as $T \to \infty$,

$$\tfrac{1}{\sqrt{T}} \sum_{t=1}^{T} \left. \frac{\partial l_t(\theta)}{\partial \alpha} \right|_{\theta=\theta_0} \xrightarrow{D} N(0, \omega_S).$$

Explain what ω_S is and how it is derived.

Hint: Note that one can show that x_t is weakly mixing if $\alpha_0 = 2$ but in this case the variance of x_t is not finite.

Question A.3: Assume that $\omega = \omega_0 = 1$ is known, and as before $\alpha_0 = 2$. It follows that,

$$\tfrac{1}{T} \sum_{t=1}^{T} \frac{\partial^2 l_t(\theta)}{\partial \alpha^2} = \tfrac{1}{T} \sum_{t=1}^{T} \left(2 \tfrac{x_t^2}{\sigma_t^2} - 1 \right) v_t^2, \quad \text{with } v_t = \tfrac{2|x_{t-1}|}{\omega + \alpha |x_{t-1}|}.$$

Use the second derivative stated to argue that as $T \to \infty$, $\sqrt{T}(\hat{\alpha} - 2)$ is asymptotically Gaussian distributed. Please be explicit about which conditions and theorems you apply for this to hold.

Question A.4: Consider the data and ACF in Figure A.1. If modelled by the model in (A.1)-(A.2) one finds a well-specified model with $\hat{\alpha} = 0.9$ and $\hat{\omega} = 0.07$.However, instead a different Gaussian ARCH was estimated and the output is as follows:

Model estimated:	
$x_t = \sigma_t z_t$ where	$\sigma_t^\delta = \omega + \alpha(\|x_{t-1}\| - \gamma x_{t-1})^\delta + \beta\sigma_{t-1}^\delta$, $\delta > 0$.
MLE and std.deviation:	
MLE	Std.dev.
$\hat{\omega} = 0.06$	0.02
$\hat{\alpha} = 0.9$	0.07
$\hat{\beta} = 0.01$	0.02
$\hat{\gamma} = 0.01$	0.03
$\hat{\delta} = 1.1$	0.15
LM-test Normality	p-val $= 0.45$

Comment on this and explain in what way, if any, the two models differ.

In particular, is it reasonable to conclude that $\beta_0 = 0$ and $\delta_0 = 1$? If so (not), what does this imply.

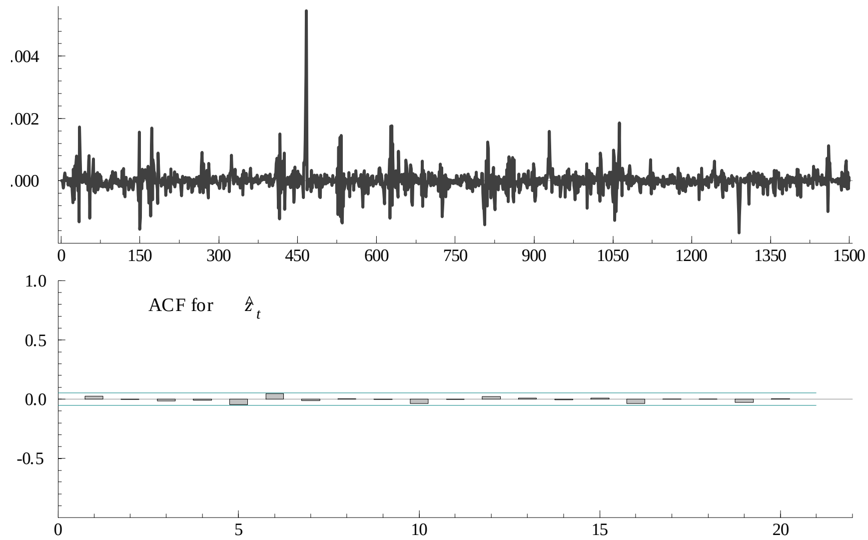

Fig. A.1: Return series x_t in top graph and ACF for absolute value of scaled residuals $\hat{z}_t$ in lower graph.

Question B:

Question B.1: As part of a discussion of "bubbles" in financial markets consider the spot price series y_t in Figure B.1 with t=1,2,...,T=1620.

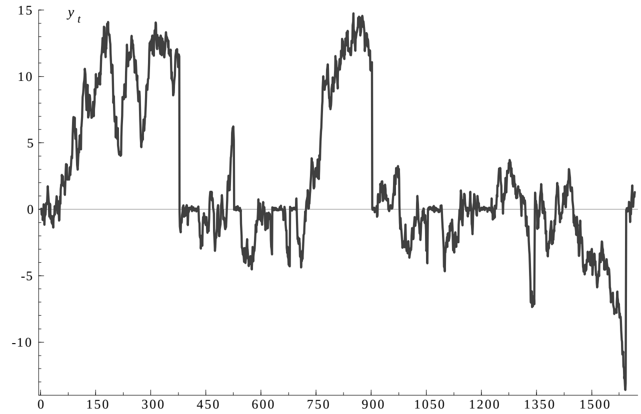

Figure B,1

For estimation the following 2-state switching model was applied:

$$y_t = \rho_{s_t} y_{t-1} + \sigma_{s_t} z_t, \quad t = 1, 2, ..., T = 1620. \tag{B.1}$$

Here z_t are iid N(0,1) distributed, and the switching variable $s_t \in \{1, 2\}$, with the switching governed by the transition matrix $P = (p_{ij})_{i,j=1,2}$. Moreover,

$$\rho_{s_t} = \rho 1 (s_t = 1) \quad \text{and } \sigma^2_{s_t} = \sigma^2_1 1 (s_t = 1) + \sigma^2_2 1 (s_t = 2). \tag{B.2}$$

Gaussian likelihood estimation gave the following output, with misspecification tests in terms of smoothed standardized residuals $\hat{z}^*_t$:

MLE of P:	$\hat{p}_{11} = 0.95$ $\hat{p}_{21} = 0.07$
MLE of ρ :	$\hat{\rho} = 0.99$
MLE of σ^2_1 and σ^2_2	$\hat{\sigma}^2_1 = 0.56$ and $\hat{\sigma}^2_2 = 0.1$
	p-values:
LM-test for Normality of $\hat{z}^*_t$:	0.12
LM-test for no ARCH in $\hat{z}^*_t$:	0.10
LR-test of $\rho = 1$:	0.81

Interpret the model. What would you conclude on the basis of the output and Figure B.1?

Question B.2: In order to find the MLE of $\theta = (p_{11}, p_{22}, \rho, \sigma_1^2, \sigma_2^2)$ the function $M(\theta)$ given by,

$$M(\theta) = \sum_{i,j=1}^{2} \log p_{ij} \sum_{t=2}^{1620} p_t^*(i,j) + \sum_{j=1}^{2} \sum_{t=2}^{1620} p_t^*(j) \log f_\theta(y_t|y_{t-1}, j), \qquad \text{(B.3)}$$

can be used. Provide an expression for $f_\theta(y_t|y_{t-1}, 1)$.

Question B.3: Explain how you would use $M(\theta)$ from (B.3) in order to find the MLE $\hat{\theta}$.

Comment on what Figure B.2 shows in relation to finding $\hat{\theta}$.

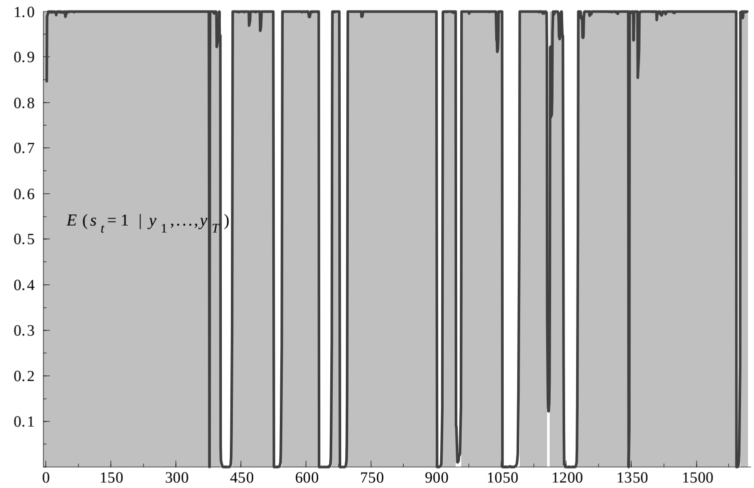

Figure B.2

Question B.4: Now assume that at time T, $s_T = 1$. In order to forecast if one will enter state 1 at $T + 2$ say, compute

$$P(s_{T+2} = 1|s_T = 1),$$

and provide an estimate of this given the output.

Question B.5: A pragmatic solution was suggested: Simply ignore the regime switching and consider a simple GARCH(1,1) model for returns Δy_t,

$$\Delta y_t = \sigma_t z_t, \text{ with } \sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$$

and z_t iid N(0, 1). We find that the QMLE of α and β satisfy $\hat{\alpha} + \hat{\beta} \approx 1$. Discuss and provide a possible explanation of this.